ToppersNotes

THE IIT-JEE SECRET
MECHANICS

Sierra Innovations Pvt. Ltd.

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In Kinematics, we study the relation b/w parameters of motion not the cause of motion.

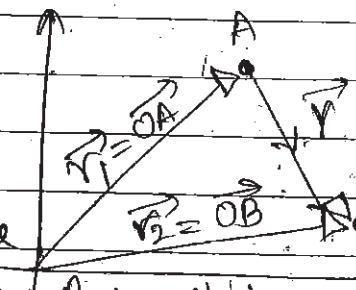
PARAMETERS

1- Position
2- Displacement
3- Distance
4- Average velocity
5- Inst. velocity
6- Avg. Speed
7- Inst. Speed
8- Acceleration

POSITION & DISPLACEMENT (vector)

→ Change in position vector is called displacement. It is directed along initial to final position.
→ Its magnitude is min. dist. b/w initial & final position
→ It is vector quantity.

किसी Particle को exist करने के लिए Cartesian Plane जरूरी है।



Any particle requires Cartesian plane for its existance

Change = Δ = final - initial

$$\Delta\vec{r} = \vec{OB} - \vec{OA}$$

$$\vec{D} = \vec{AB} \quad (\because \vec{AB} = \vec{OB} - \vec{OA})$$

यह जरूरी नहीं होता है कि displacement वाली लाईन ही particle का path हो।
→ Its not necessary that displacement line is only the path followed by particle

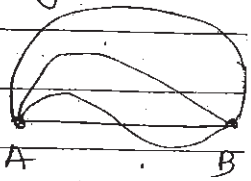
DISTANCE (Scalar)

Length of path travelled is known as distance.

Distance b/w two points A & B can have many values. any value नहीं हो सकती है। Bcoz

$$\text{Distance} \geq |\text{Displacement}|$$

disp. से कम नहीं हो सकती है।
distance cant be less than displacement



Many paths Can be travelled

* { v_{avg} = average velocity
  v_{avg} = average speed.

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AVERAGE & INST. VELOCITY (vector)

Average velocity is the ratio of total disp. and time taken.

$$\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{D}}{t}$$

Δ = Change
x = Position
$\Delta \vec{x}$ = disp.

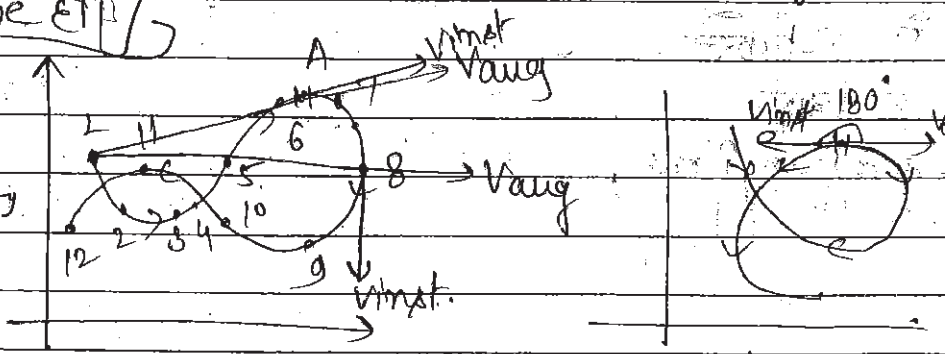
→ Velocity is not defined at a single pt. We need min. two pt. to define velocity.

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

distance & time interval are very small

→ यह जरूरी नहीं है कि inst. & avg. velocity का magnitude हमेशा same हो।

It's not necessary that magnitude of inst. and avg velocity always remain same.



v_{inst} and v_{avg} are same at pt. 'A' and v_{inst} and v_{avg} are mutually ⊥ at point '8'

but direction may be same.

→ Inst. velocity is rate of change of disp. vector w.r.t. time.

Direction of velocity will be diff. with
" " accel.

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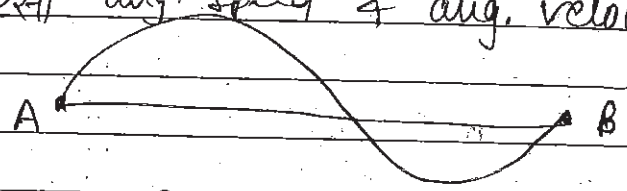
AVERAGE SPEED & INST. SPEED

Average speed is the ratio of total distance travelled & time taken.

$$\boxed{V_{avg} \text{ (avg. speed)} = \frac{Dist}{Time}}$$

Instantaneous speed is the magnitude of inst. velocity
But ऐसा avg. speed & avg. velocity के साथ नहीं होगा



A & B के बीच के curve को time से divide करेंगे तो avg. speed मिलेगी but AB st. line को time से divide करने पर avg. velocity मिलेगी (bcoz for avg velocity we use $\frac{disp}{time}$ and disp is st. line AB)

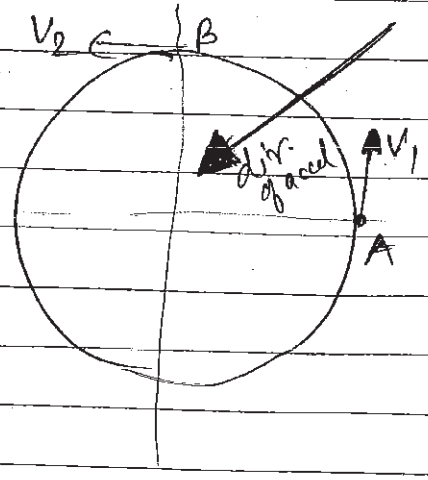
$$\boxed{\left(V_{avg}^{speed}\right)_{min} = \left|\vec{v}_{avg\,(velo.)}\right|}$$

If the curve between A and B is divided by time, then we get avg speed, but if AB st. line is divided by time then we get avg. Velocity

ACCELERATION

Rate of change of velocity vector w.r.t. time is known as acceleration. It is directed along the change in velocity, not in the direction of velocity.

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$



$v = 4 m/sec$
Time A to B = 2 sec

$V_1 = 4\hat{j}$ $\quad V_2 = -4\hat{i}$

$$\vec{a} = \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{V_2} - \vec{V_1}}{2}$$

$$= \frac{-4\hat{i} - (4\hat{j})}{2} = -2\hat{i} - 2\hat{j}$$

Now draw it to find dir. of accel.

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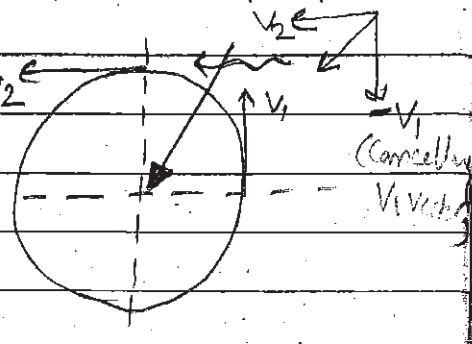
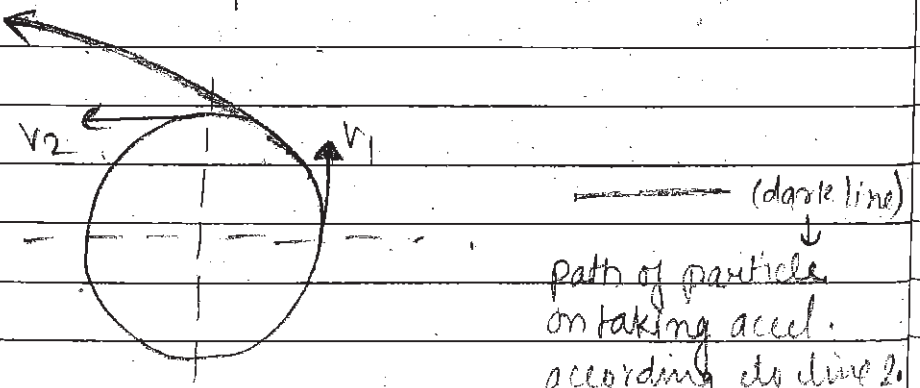
R In previous question, the direction of acceleration is calculated by drawing the resultant vector of two vectors.

But

* यहाँ पर एक doubt आ सकता है कि हम accel. का position line 1 (red) की जगह line 2 (blue) क्यूँ नहीं लिया ?



Ans:- अगर हम accel. की direction line 2 लेते तो velocity v_1 का कम्पोनेन्ट जो line 1 से produce हो रहा था तो वो v_1 के opposite direction में होने की वजह से उसे 0 (zero) कर रहा था जो कि acceleration के लिए जरूरी था। but अगर हम line 2 को accel. की direction लेते हैं तो v_1 zero नहीं होगा and v_2 भी नहीं जिसकी वजह से particle अपने पथ से थोड़ा deviate हो जाएगा।



* A doubt can occur why we can't accelration position on line 2 instead of line 1 ?

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JEE

Ex

R

$\frac{d|\vec{v}|}{dt}$ (I) $\left|\frac{d\vec{v}}{dt}\right|$ (II)

Is any case there for wh

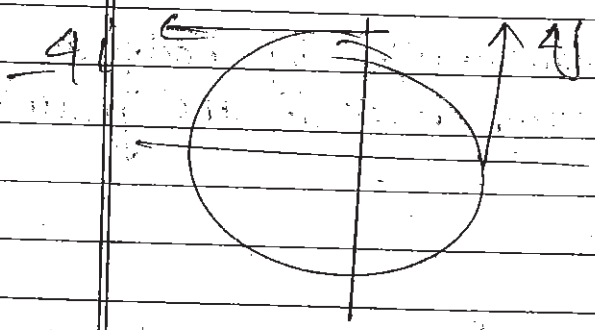
(A) $I = 0$, $II \neq 0$

(B) $I \neq 0$, $II = 0$.

Sol $\frac{d|\vec{v}|}{dt}$ $|\vec{v_1}|, |\vec{v_2}|$ (so) A can exist but B can't

$$\frac{d|\vec{v}|}{dt} = \frac{|\vec{v_2}| - |\vec{v_1}|}{dt}$$

for this let we take example.



$t = 2s$.

(I) $|\vec{v_1}| = 4$

$|\vec{v_2}| = 4$.

$$\frac{|\vec{v_2}| - |\vec{v_1}|}{dt} = \frac{4-4}{2} = 0.$$

(II) $|-4i - 4j| = \frac{|d\vec{v}|}{|dt|}$

$$= \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Means if (I) $= 0$, (II) may not be zero.

If $II = 0$. $|\vec{v_2} - \vec{v_1}| = 0$ bcoz $dt \neq 0$ (never)

$\vec{v_2} = \vec{v_1} \Rightarrow |\vec{v_2}| - |\vec{v_1}| = 0$

Means If $II = 0$ [illegible]

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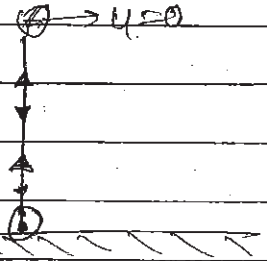
Distance travelled in nth second

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

FREE - FALL

Motion under gravity only is called free fall.

(1) Body के ऊपर जाने के लिए $t = \frac{u}{g}$ time लिया है + वापस



आने के लिए भी same time लेगी।

$t = \frac{u}{g(\text{जाने})} + \frac{u}{g(\text{आने})}$

$$t = \frac{2u}{g}$$

Time taken by body to reach back to the ground

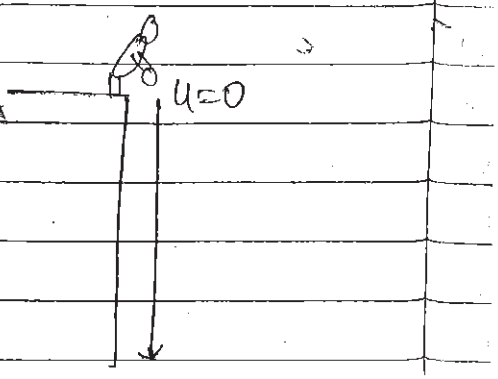
(2) Maximum height reached by ball when it is thrown with 'u' velocity

$$h_{max} = \frac{u^2}{2g}$$

(3)

$$t = \sqrt{\frac{2h}{g}}$$

Time after which ball reach to the ground



Velocity & accel. opposite dir. में ⇒ Speed slows
Velocity & accel same dir. → Speed ↑

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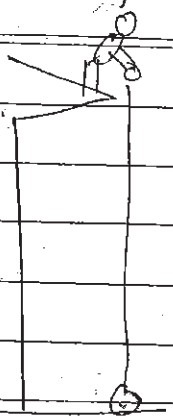
(4)

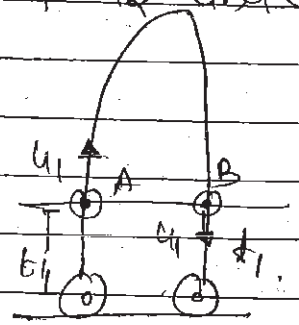


$$V = \sqrt{2gH}$$

Velocity of ball when it reaches to ground.

** (5) **

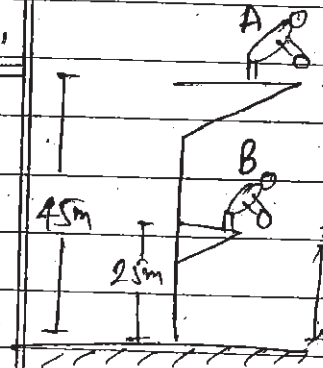
अगर velocity u से कोई बॉल फेंकी गई and उसके जाने और आने वाले दोनों path से अगर हमने एक line draw की see fig. then velocity at the pt. at which this line is cutting the path will same ⇒ time interval will also same.



velocity at pts. A & B will be same.

Ex.

R.



Man A is ready to throw the ball and when this ball is reached do the man 'B' He will also throw his ball. Ball of man A will reach the ground first. Then find the time duration Δt after which the ball of man B will reach to ground.

$t = \sqrt{\frac{2y}{g}} \Rightarrow t = \sqrt{\frac{2\times 25}{g}}$

$= \sqrt{5}$ sec

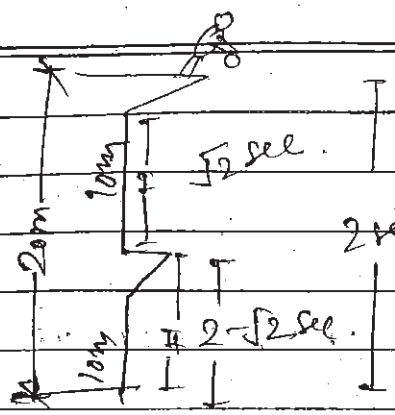
$t = \sqrt{\frac{2\times 45}{10}}$

$= 3$ sec. $\quad \Delta t = (\sqrt{5}-1)$ sec

$= 2$ sec $+ 1$ sec

Ball will reach to the man B after 2 seconds and then B will also throw his ball.

R. Ex.



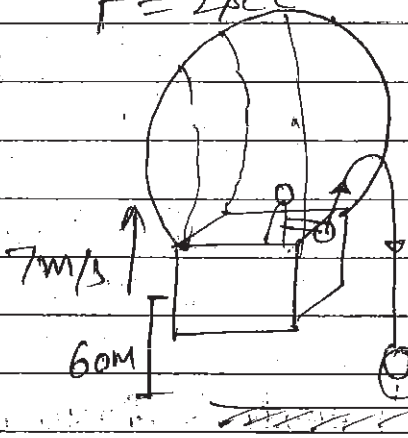
ratio of find time spent in Ist & IInd Half $\frac{t_I}{t_{II}}$

$$t = \sqrt{\frac{2\times 20}{10}} \qquad t = \sqrt{\frac{2\times 10}{10}}$$

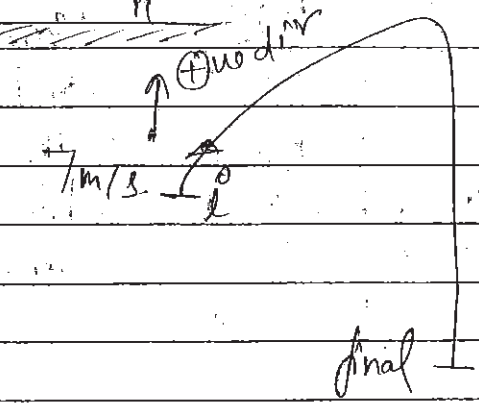
$t = 2$ sec.

$$\frac{t_I}{t_{II}} = \frac{\sqrt{2}}{2-\sqrt{2}}$$

R. Ex.



Situation is given in fig. find the time after which ball will reach to the ground.



$$S = ut + \frac{1}{2}at^2$$

$$-60 = 7t + \frac{1}{2}(-10)t^2$$

$$-60 = 7t - 5t^2$$

$$5t^2 - 7t - 60 = 0$$

$$\boxed{t = 4.3 \text{ sec}}$$

OR