



IIT - JEE

JEE MAIN & ADVANCED

NATIONAL TESTING AGENCY

भौतिक विज्ञान

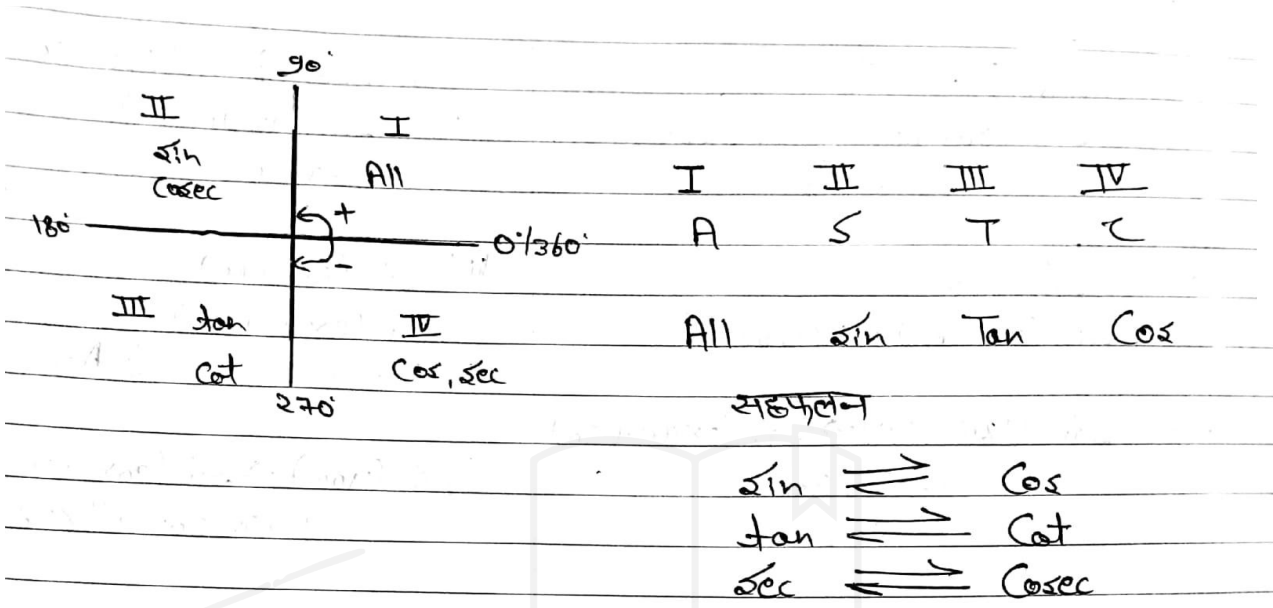
भाग - 1



विषय सूची

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फलन, ग्राफ, त्रुटिया



१. कोण \rightarrow $n\pi \neq 0$ यदि सम संख्या है तो फलन अपरिवर्तित रहेगा
 या $n \times 90^\circ \pm \theta$

२. कोण \rightarrow $\frac{n\pi}{2} \neq 0$ यदि n विषम संख्या है तो फलन वै सहफलन में बदले
 या $n \times 90^\circ \pm \theta$

Ex $\sin(-\theta) \Rightarrow -\sin\theta$
 $\cos(-\theta) \Rightarrow \cos\theta$
 $\tan(-\theta) \Rightarrow -\tan\theta$

$\rightarrow \tan\theta(-60) \Rightarrow -\tan 60 \Rightarrow -\sqrt{3}$

$\rightarrow \cos(-30) \Rightarrow \cos 30 = \frac{\sqrt{3}}{2}$

$\rightarrow \sin(90+\theta) \Rightarrow +\cos\theta$

$$\begin{aligned} \sin(60^\circ) &\rightarrow \frac{\sqrt{3}}{2} \\ \rightarrow \sin(90-60) &\Rightarrow \cos 30^\circ \\ &\rightarrow \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \rightarrow \sin(150^\circ) \\ \sin(90+60) &= +\cos 60^\circ \rightarrow \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \rightarrow \sin(180-30) &\rightarrow \sin(2 \times 90 + 30) \\ &\rightarrow -\cos 30^\circ \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \rightarrow \cos(210^\circ) \\ \text{(i) } \cos(180+30) & \\ \cos(2 \times 90 + 30) &= -\cos 30^\circ \\ &\rightarrow -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \cos(270-60) & \\ \cos(3 \times 90 - 60) &\rightarrow -\sin 60^\circ \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \rightarrow \sin(330) &\Rightarrow \sin(360-30) \\ \text{(i) } &\Rightarrow \sin(4 \times 90 - 30) \\ &= -\sin 30^\circ = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \sin(270+60) & \\ \sin(3 \times 90 + 60) & \\ \rightarrow -\cos 60^\circ &= -\frac{1}{2} \text{ Ans} \end{aligned}$$

$$\begin{aligned} \rightarrow \sin(480) &\Rightarrow \sin(90 \times 5 + 30) \\ &+ \cos 30^\circ \rightarrow \frac{\sqrt{3}}{2} \text{ Ans} \end{aligned}$$

$$\begin{aligned} \rightarrow \sin(750) &\Rightarrow \sin(90 \times 8 + 30) \\ &\rightarrow \sin 30^\circ \rightarrow \frac{1}{2} \text{ Ans} \end{aligned}$$

$$\begin{aligned} \rightarrow \tan(1140) &\rightarrow \tan(90 \times 12 + 60) \\ \tan 60^\circ &\rightarrow \sqrt{3} \end{aligned}$$

$$\begin{aligned} \rightarrow \sin(270) &\rightarrow \sin(3 \times 90 + 0) \\ &= -\cos 0^\circ \rightarrow -1 \end{aligned}$$

समाकलन (Integration)

1 अनिश्चित समाकलन

$$\text{(i) } \int x^n dx \Rightarrow \frac{x^{n+1}}{n+1} + C \quad \text{जहाँ } C = \text{नियतांक } n \neq -1$$

$$\begin{aligned} \text{(ii) } \int x^{-1} dx &= \int \frac{1}{x} dx = \log x + C \quad n = -1 \\ &= \ln x + C \end{aligned}$$

Ex ① $\int x^6 dx$ -
 $\rightarrow \frac{x^{6+1}}{6+1} + C$
 $\frac{x^7}{7} + C$ Ans

② **माझ्या समाकलन**
 $\rightarrow \int_2^3 x^2 dx$
 Upper limit 3
 Lower limit 2

Ex ② $\int \sqrt{x} \cdot dx$
 $\int x^{1/2} dx$

Solution $\rightarrow \left[\frac{x^{2+1}}{2+1} \right]_2^3$

$\frac{x^{1/2+1}}{1/2+1} + C$

$\left[\frac{x^3}{3} \right]_2^3 \rightarrow \left[\frac{3^3}{3} - \frac{2^3}{3} \right]$

$\frac{x^{3/2}}{3/2} + C \rightarrow \frac{2}{3} x^{3/2} + C$

$\left[9 - \frac{8}{3} \right] = \frac{19}{3}$ Ans

Ex ③ $\int (x^3 + x^2) dx$
 $\int x^3 dx + \int x^2 dx + C$

Ex ① $\int_3^5 x^7 dx$

$\int \frac{x^{3+1}}{3+1} + \int \frac{x^{2+1}}{2+1} + C$

$\left[\frac{x^{7+1}}{7+1} \right]_3^5$

$\int \frac{x^4}{4} + \int \frac{x^3}{3} + C$

$\left[\frac{x^8}{8} \right]_3^5 \rightarrow \left[\frac{(5)^8}{8} - \frac{(3)^8}{8} \right]$

Ans

Ex ④ $\int \sqrt[3]{x} dx + C$
 $\int \sqrt[3]{x} dx + C$
 $\int x^{1/3} dx + C$
 $\int \frac{x^{1/3+1}}{1/3+1} dx + C$

$\frac{390625}{8} - \frac{6561}{8}$

$\int \frac{x^{4/3}}{4/3} \rightarrow \int \frac{3}{4} x^{4/3}$ Ans

$\frac{384064}{8} \rightarrow 48008$ Ans

$$\text{Ex ③} \int_a^b \frac{1}{x} dx$$

$$= \left[\log x \right]_a^b$$

$$[\log b - \log a] \text{ Ans}$$

$$\text{Ex ⑤} \int_2^3 (x^3 + x) dx$$

$$\int_2^3 x^3 dx + \int_2^3 x^1 dx + C$$

$$\int_2^3 \frac{x^{3+1}}{3+1} dx + \int_2^3 \frac{x^{1+1}}{1+1} dx + C$$

$$\text{Ex ④} \int_3^5 \frac{1}{x^{-4}} dx + C$$

$$\int_3^5 \frac{x^4}{4} dx + \int_2^3 \frac{x^2}{2} dx + C$$

$$\int_3^5 x^4 dx + C$$

$$\left[\frac{(3)^4}{4} - \frac{(2)^4}{4} \right] + \left[\frac{(3)^2}{2} - \frac{(2)^2}{2} \right] + C$$

$$\left[\frac{x^{4+1}}{4+1} \right]_3^5$$

$$\left[\frac{81}{4} - \frac{16}{4} \right] + \left[\frac{9}{2} - \frac{4}{2} \right] + C$$

$$\left[\frac{x^5}{5} \right]_3^5$$

$$\left[\frac{81-16}{4} \right] + \left[\frac{9-4}{2} \right] + C$$

$$\left[\frac{(5)^5}{5} - \frac{(3)^5}{5} \right]$$

$$\left[\frac{65}{4} \right] + \left[\frac{5}{2} \right] + C$$

$$\frac{625 - 243}{5}$$

$$\frac{65 + 10}{4} = \frac{75}{4} \text{ Ans}$$

$$\frac{382}{5} \text{ Ans}$$

Ex 6) $\int_1^4 (\sqrt{x} + \frac{1}{\sqrt{x}}) dx + c$

$$\int_1^4 (x)^{\frac{1}{2}} + \int_1^4 (x)^{-\frac{1}{2}}$$

$$\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_1^4 + \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_1^4$$

$$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 + \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^4$$

$$\left[\frac{2^{\frac{3}{2}} - 1^{\frac{3}{2}}}{\frac{3}{2}} \right] + \left[\frac{2^{\frac{1}{2}} - 1^{\frac{1}{2}}}{\frac{1}{2}} \right]$$

Ans

Ex 7) $\int_2^4 (x + \frac{1}{x}) dx$

$$\int_2^4 x^1 + \int_2^4 x^{-1} dx +$$

$$\left[\frac{16-4}{2} \right] + \log \frac{4}{2}$$

$$\int_2^4 \frac{x^{1+1}}{1+1} + \int_2^4 \log x$$

$$\frac{12}{2} + \log^2$$

$$\int_2^4 \frac{x^2}{2} + \left[\log x \right]_2^4$$

$$6 + \log^2$$

$$\left[\frac{4^2}{2} - \frac{2^2}{2} \right] + \left[\log 4 - \log 2 \right]$$

$$\left[\frac{16}{2} - \frac{4}{2} \right] + \log \frac{4}{2}$$

अनिश्चित समाकलन के उदा

Ex 1. $\int x^6 \cdot dx$

$$\int \frac{x^{6+1}}{6+1} dx + C$$

$$\int \frac{x^{6+1}}{6+1} dx + C$$

$$\frac{x^7}{7} + C$$

Ex 4. $\int (x^3 + x^2) dx$

$$\int (x^3) dx + \int (x^2) dx$$

$$\int \frac{x^{3+1}}{3+1} dx + \int \frac{x^{2+1}}{2+1} dx + C$$

$$\int \frac{x^4}{4} dx + \int \frac{x^3}{3} dx + C \quad \text{Ans}$$

Ex 2. $\int \sqrt{x} \cdot dx$

$$\int x^{1/2} \cdot dx$$

$$\int \frac{x^{1/2+1}}{1/2+1} dx + C$$

$$\int \frac{x^{3/2}}{3/2} dx + C$$

$$\int \frac{2}{3} x^{3/2} dx + C$$

$$\int \frac{2}{3} x^{3/2} dx + C$$

निश्चित समाकलन

Ex 5. $\int_3^5 x^7 dx$

$$\left[\frac{x^{7+1}}{7+1} \right]_3^5$$

$$\left[\frac{x^8}{8} \right]_3^5$$

$$\left[\frac{x^8}{8} \right]_3^5$$

$$\left[\frac{(5)^8}{8} - \frac{(3)^8}{8} \right] \quad \text{Ans}$$

Ex 3. $\int \sqrt[3]{x} dx$

$$\int x^{1/3} dx$$

$$\int \frac{x^{1/3+1}}{1/3+1} dx + C$$

$$\int \frac{x^{4/3}}{4/3} dx + C \rightarrow \int \frac{3}{4} x^{4/3} dx + C \quad \text{Ans}$$

$$\textcircled{1} \int \cos ax \, dx = \frac{\sin ax}{a} + C$$

$$\text{Ex 6.} \quad \textcircled{2} \int \sin ax \, dx = -\frac{\cos ax}{a} + C$$

Formula's

$$\textcircled{3} \int \sin \theta \cdot d\theta = -\cos \theta + C \quad \left. \begin{array}{l} \textcircled{3} \\ \textcircled{4} \end{array} \right\} \text{अनिश्चित समाकलन के}$$

$$\textcircled{4} \int \cos \theta \cdot d\theta = \sin \theta + C$$

~~निश्चित समाकलन~~

$$\text{Ex 1} \quad \int_0^{\pi/2} \sin \theta \cdot d\theta = -[\cos \theta]_0^{\pi/2}$$

$$- [\cos \theta]_0^{\pi/2}$$

$$- [\cos \frac{\pi}{2} - \cos 0]$$

$$- [0 - 1] \rightarrow 1 \text{ An}$$

$$\text{Ex 2} \quad \int_0^{\pi} \cos \theta \cdot d\theta$$

$$[\sin \theta]_0^{\pi}$$

$$\sin 180^\circ - \sin 0$$

$$[0 - 0]$$

$$= 0 \text{ An}$$

$$\cos \pi \text{ या } \cos 180^\circ \rightarrow -1$$

$$\sin \pi \text{ या } \sin 180^\circ \rightarrow 0$$

$$\cos 2\pi \text{ या } \cos 360^\circ \rightarrow 1$$

$$\sin 2\pi \text{ या } \sin 360^\circ \rightarrow 0$$

• गुणोत्तर श्रृंखला

$$a, ar, ar^2, ar^3 \dots ar^n$$

$a \rightarrow$ प्रथम पद $r =$ सर्व अंतर

$$n \text{ पदों का योग} \rightarrow \begin{cases} \frac{a(r^n - 1)}{r - 1} & \text{या} & \frac{1 - r^n}{1 - r} \\ (r > 1) & & (r < 1) \end{cases}$$

Ex 1, 4, 16, 64 ... 10 पदों तक योग

$$r = \frac{64}{16} \rightarrow 4 \quad n = 10$$

$$S_{10} = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} \rightarrow \frac{1((4)^{10} - 1)}{4 - 1}$$

$$S_{10} \rightarrow \frac{1048576 - 1}{3}$$

$$S_{10} \rightarrow \frac{1048575}{3}$$

$$S_{10} \rightarrow 349525$$

Very most Formula यदि $n = \infty$ तब $r < 1$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_{\infty} = \frac{a(1 - r^{\infty})}{1 - r}$$

① $\left. \begin{matrix} r < 1 \text{ होने पर} \\ r^{\infty} = 0 \end{matrix} \right\}$

$$S_{\infty} \rightarrow \frac{a(1 - 0)}{1 - r}$$

$$S_{\infty} = \boxed{\frac{a}{1 - r}}$$

Ex $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots \infty$ तक

$a = 1$

$r = \frac{1}{2}$

$$S_{\infty} = \frac{1}{1 - \frac{1}{2}} \rightarrow \frac{1}{\frac{1}{2}} = 2A$$

Formula यदि

Ex $n = \infty$ तथा $r > 1$

$$S_{\infty} = \frac{a(r^{\infty} - 1)}{r - 1}$$

$r > 1$ होने पर $r^{\infty} = \infty$ होगा

$$S_{\infty} = \frac{a(\infty - 1)}{r - 1}$$

$$S_{\infty} = \infty \text{ (v. most)}$$

Ex $1 + 2 + 4 + 8 + 16 \dots \infty$ तक

$a = 1$ $r = 2$

$r > 1$ $n = \infty$

अतः $S_{\infty} = \infty$

ग्राफ :->

1. सरल रेखा :->

$$y = mx + c$$

$m =$ ढाल / प्रवणता / झुकाव

$$m = \tan \theta$$

$c \Rightarrow$ अंतः खण्ड

m तथा y होने पर ग्राफ सरल रेखा बनेगा

यदि $m = +$ तो + वाले में
 तथा $m = -$ हो तो - वाले में

Ex ① $y = 5x$

Ans Step I \rightarrow सर्वप्रथम $x = 0$ रखने पर

$$y = 5 \times 0$$

$$y = 0$$

निर्देशांक $(0, 0)$

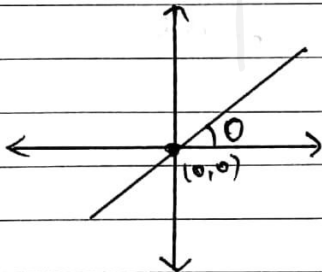
Step II $y = 0$ रखने पर

$$y = 5x$$

$$0 = 5x$$

$$x = 0$$

निर्देशांक $(0, 0)$



$$m = 5$$

$$\tan \theta = 5$$

Ex ② $y = -5x$

Step I सर्वप्रथम $x = 0$

$$y = -5 \times 0 = 0$$

$$y = 0$$

$$(0, 0)$$

$$m = -5$$

$$m = \tan \theta$$

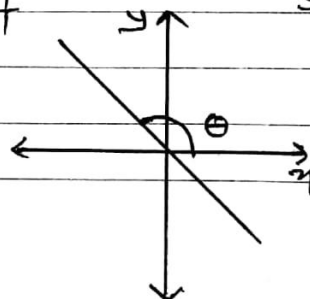
$$\tan \theta = -5$$

Step II $y = 0$ रखेंगे

$$y = -5x$$

$$0 = x$$

$$(0, 0)$$



Ex 3. $y = -x + 3$

$m = -1$

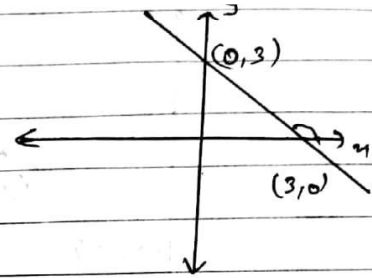
Step I अवधि में $x = 0$

$\text{slope} = -1$

$y = 3$ $(0, 3)$

Step II $y = 0$

$x = 3$ $(3, 0)$



Ex 4 $2y = x + 3$

Step I $x = 0$

$2y = 3$

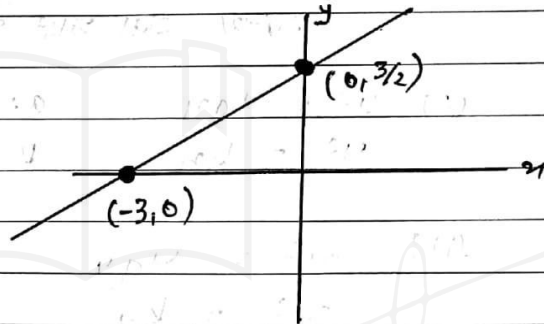
$y = 3/2$ $(0, 3/2)$

Step II $y = 0$

$0 = x + 3$

$x = -3$

$(-3, 0)$



Ex 5 $2y = x + 3$

यहाँ ग्राफ का ढाल बताओ

$y = mx + c$

$y = \frac{x + 3}{2}$

$y = \frac{x}{2} + \frac{3}{2}$

$y = mx + c$ से तुलना करने पर

$m = \frac{1}{2}$

Ex 6 $x = 2y + 6$

का ग्राफ बनाओ तथा इसके ढाल भी बताओ

Step I $x = 0$ रखने पर

Step II $y = 0$ रखने पर

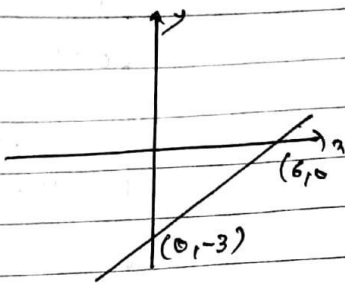
$0 = 2y + 6$

$x = 6$

$y = -\frac{6}{2}$

$(6, 0)$

$y = -3$ $(0, -3)$



दाता

$$y = mx + c$$

$$\frac{x-6}{2} = y$$

$$\frac{x}{2} - \frac{6}{2} = y$$

$$m = \frac{1}{2} \text{ Ans}$$

② परवलय \rightarrow जब x^2 या फिर y^2 हो तब ग्राफ परवलय होगा

परवलय उस अक्ष के दोनों तरफ बनेगा जिसकी घात 1 है

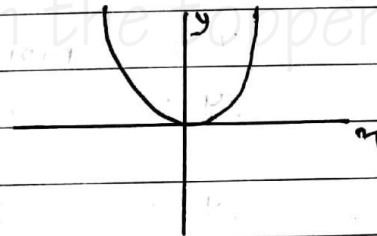
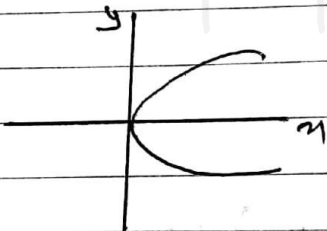
(i) $x^2 = 4ay$ $a =$ निम्तांक (निम्ता)

$x^2 = ky$ $k =$ निम्तांक

(ii) $x^2 = 4ay$
 $x^2 = ky$

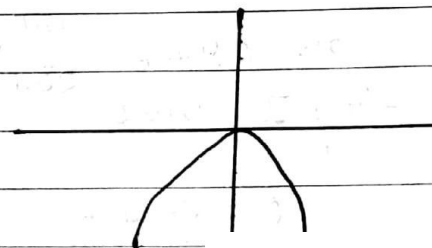
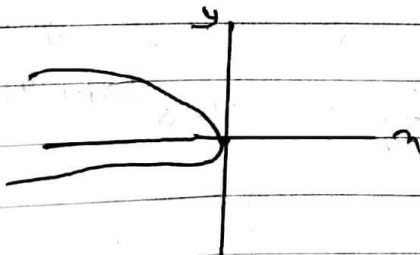
(i) का ग्राफ

(ii) का ग्राफ



(iii) $y^2 = -4ax$
 $y^2 = -kx$

(iv) $x^2 = -4ay$
 $x^2 = -ky$



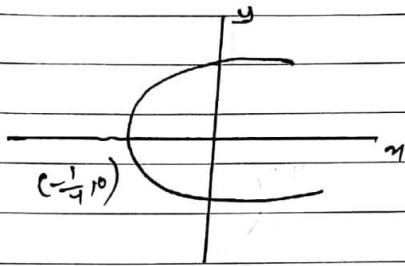
Ex ① $y^2 = 8x + 2$

$y = 0$ पर

$8x = -2$

$x = \frac{-2}{8} = -\frac{1}{4}$

$(-\frac{1}{4}, 0)$



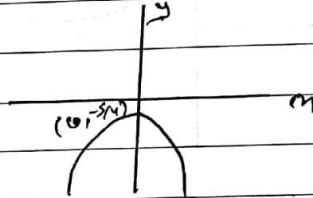
Ex ④ $x^2 + 4y + 5 = 0$

$x^2 = -4y - 5$

$x = 0$ पर

$y = -5/4$

$(0, -5/4)$



Ex $E = \frac{1}{2}mv^2$

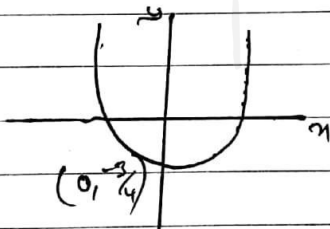
Ex ② $x^2 = 4y + 3$

$x = 0$ पर

$0 = 4y + 3$

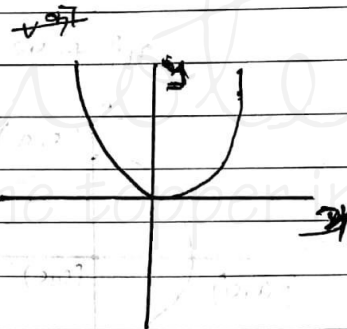
$y = -3/4$

$(0, -3/4)$



$\frac{2E(y)}{m} = v^2(x)$

$(0, 0)$ से गुजरता

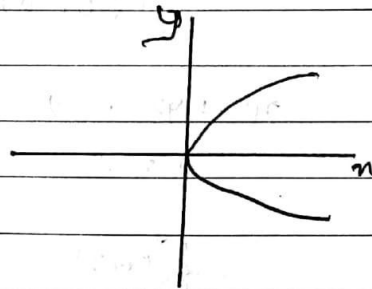
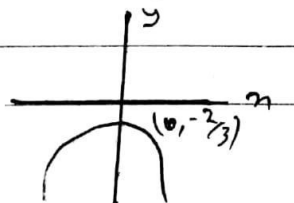


Ex ③ $-x^2 = 3y + 2$

$x^2 = -3y - 2$

$x = 0$ पर

$y = -2/3$ $(0, -2/3)$



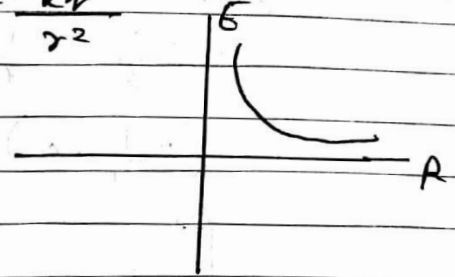
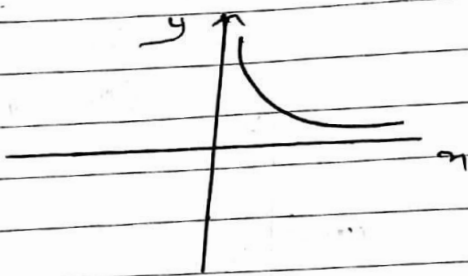
③ अतिपरवलय

$$y = \frac{c}{x^2}$$

$c =$ निरंतरांक

Ex $E = \frac{kV}{r^2}$

E व r में उल्टे संबंध



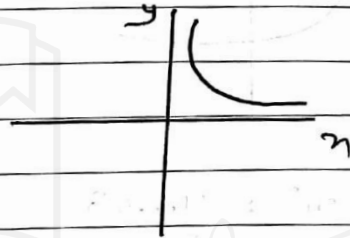
④ आयताकार अतिपरवलय

$$xy = c$$

या

$$y = \frac{c}{x}$$

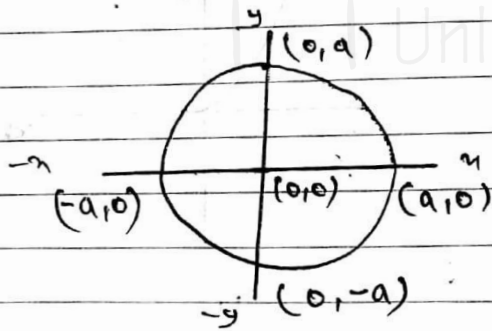
$$x = \frac{c}{y}$$



⑤ वृत्त

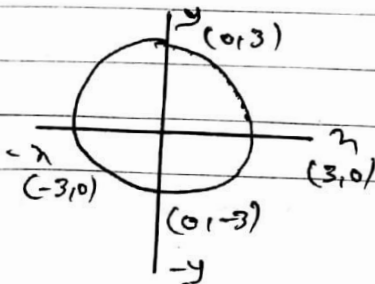
$$x^2 + y^2 = a^2$$

(वृत्त की त्रिज्या = a)



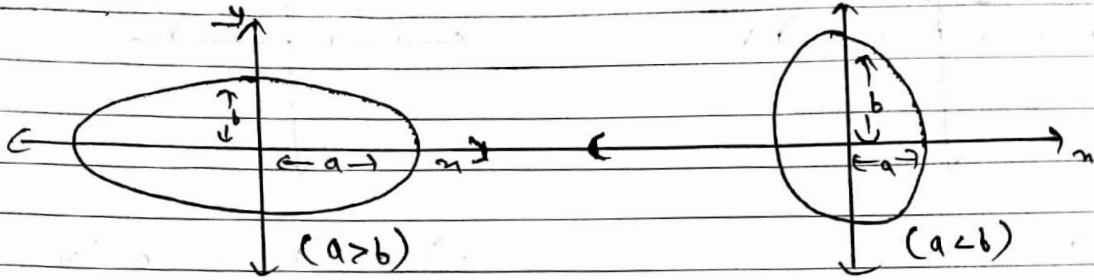
Ex $x^2 + y^2 = 9$

$$a = 3$$

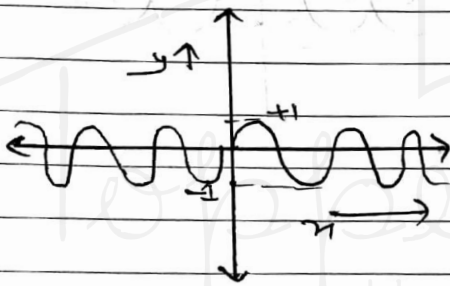


⑥ दीर्घवृत्त

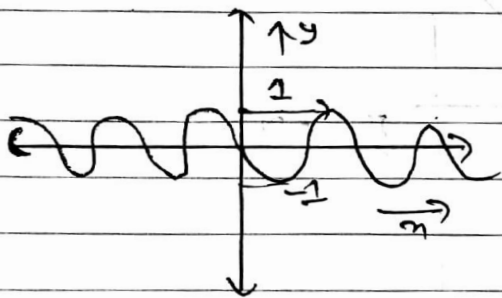
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



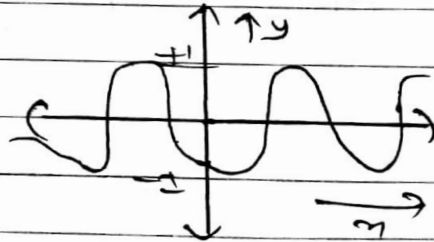
→ $y = \sin x$ का ग्राफ
 (-1 से +1 तक मान होता है)



→ $y = -\sin x$



→ $y = -\cos x$



→ $y = \cos x$

